

CONSTRUCTING A COMPOSITE FINANCIAL INCLUSION INDEX FOR BRAZIL CONSTRUINDO UM ÍNDICE COMPOSTO DE INCLUSÃO FINANCEIRA PARA O BRASIL

CONSTRUCCIÓN DE UN ÍNDICE DE INCLUSIÓN FINANCIERA COMPUESTO PARA BRASIL

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ABSTRACT

Study objective: This work aimed to develop a composite index of financial inclusion for the Brazilian territory.

Methodology/approach: The Displaced Ideal Method was used to operationalize the data.

Originality/relevance: an inclusive financial system necessarily involves the democratization of finance. Financial inclusion represents a multidimensional social phenomenon that is characterized by the availability, access and use by people and companies of the products and services offered by financial institutions.

Main results: the results show that, although the financial system in Brazil is accessible and available, the use by people and companies of its products and services is still limited, something seen as a barrier for the potential of finance to promote the broader social and economic development.

Contributions to knowledge: it essentially consists of presenting a research frontier, endowed with contributions in a state of scientific turmoil and that therefore deserves to receive new studies that envision new perspectives, as well as confirmations and projections of new scenarios.

Technical and corporate contributions: the study offers a consistent analysis of a model that can be applied to decisions, both in terms of public and corporate policies.

Keywords: Financial system, Financial inclusion, Composite index, Displaced Ideal Method, Brazil.

RESUMO

Objetivo do trabalho: este trabalho teve como objetivo desenvolver um índice composto de inclusão financeira para o território brasileiro.

Metodologia/Abordagem: recorreu-se ao Displaced Ideal Method para a operacionalização dos dados.

Originalidade/Relevância: um sistema financeiro inclusivo passa, necessariamente, pela democratização das finanças. A inclusão financeira representa um fenômeno social multidimensional que se caracteriza pela disponibilidade, acesso e uso por parte de pessoas e empresas aos produtos e serviços oferecidos por instituições financeiras.

Principais resultados: os resultados mostram que, apesar de o sistema financeiro no Brasil apresentar-se acessível e disponível, o uso por parte de pessoas e empresas dos seus produtos e serviços ainda é limitado, algo entendido como uma barreira para que o potencial das finanças consiga promover o desenvolvimento social e econômico mais abrangente.

Contribuições ao conhecimento: consiste, essencialmente, em apresentar uma fronteira de pesquisas, dotada de contribuições em estado de turbulência científica e que merece, portanto, receber novos estudos que vislumbrem novas perspectivas, assim como confirmações e projeções de novos cenários.

Contribuições técnicas e corporativas: o estudo oferece uma análise consistente de modelo que pode ser aplicável às decisões, tanto no que tange às políticas públicas, quanto às corporativas.

Palavras-chave: Sistema financeiro, Inclusão financeira, Indice composto, Displaced Ideal Method, Brasil.



RESUMEN

Objetivo del trabajo: este trabajo tuvo como objetivo desarrollar un índice compuesto de inclusión financiera para el territorio brasileño.

Metodología/Enfoque: se utilizó el Método del Ideal Desplazado para operacionalizar los datos.

Originalidad/Relevancia: un sistema financiero inclusivo pasa necesariamente por la democratización de las finanzas. La inclusión financiera representa un fenómeno social multidimensional caracterizado por la disponibilidad, acceso y uso por parte de personas y empresas de los productos y servicios que ofrecen las instituciones financieras.

Principales resultados: los resultados muestran que, a pesar de que el sistema financiero en Brasil es accesible y disponible, el uso de sus productos y servicios por parte de personas y empresas aún es limitado, algo entendido como una barrera al potencial de las finanzas para promover desarrollo social y económico más amplio.

Contribuciones al conocimiento: consiste esencialmente en presentar una frontera de investigación, dotada de aportes en un estado de turbulencia científica y que por tanto merece recibir nuevos estudios que vislumbren nuevas perspectivas, así como confirmaciones y proyecciones de nuevos escenarios.

Contribuciones técnicas y corporativas: el estudio ofrece un análisis consistente de un modelo aplicable a las decisiones, tanto en materia de políticas públicas como corporativas.

Palabras clave: Sistema financiero, Inclusión financiera, Índice compuesto, Método Ideal Desplazado, Brasil

1. INTRODUCTION

Financial exclusion is a type of social exclusion, and initially focused on the issue of geographic (territorial) access to the formal financial system, in particular, to banking establishments. Later, it came to be seen as the inability of agents to access and effectively use financial products and services (Carbo, S., Gardener, E. and Molyneux, 2007; Leyshon & Thrift, 1995).

Leyshon (2009) pointed out that financial exclusion can be understood as the process by which people and companies, usually of low and moderate income, are directly or indirectly excluded from the formal financial system.

Financial products and services provide customers with facilities in the form of payment methods, savings accounts, credit and investment opportunities. People and companies that do not have access to these products and services are financially excluded and face various restrictions to improve their well-being (Simpson & Buckland, 2009).

Financial exclusion can also be understood as a multidimensional phenomenon, as it encompasses a whole complex of barriers to the availability, access and use of financial Revista Gestão & Tecnologia, Pedro Leopoldo, v. 22, n.1, p. 168-192, jan./mar.2022



products and services. Availability and access are essentially related to the supply side; use is already related to the supply and demand side (World Bank, 2014).

Therefore, financial exclusion is a term used to define the processes that serve to prevent certain social groups and individuals from having access to and use of the institutions that make up the formal financial system (Gloukoviezoff, 2007). The problem of financial exclusion is more serious for people in difficult economic situations and small and medium-sized companies, but it is not limited to these groups (Demirgüç-Kunt et al., 2008).

There are many reasons to believe that financial institutions are very useful. However, the infrastructure of the financial system has not yet brought the desired harmonious society. The democratization of finance should reduce the randomness of our lives, but for the system to work well, it is necessary to further develop its intrinsic logic, and this requires an improvement in its nature and extent, including promoting knowledge essential for understanding its operation (Shiller, 2013a).

The issue of financial inclusion, defined by this term, was first brought to the fore in world history in 1997, in the United Kingdom, when it was suggested that an individual should be considered as not excluded from the financial system as far as they can, via formal institutions, manage your daily transactions, meet predictable and some unpredictable expenses, as well as manage a loss of income and avoid or reduce problematic debt (Mitton, 2008).

In many cases, financial inclusion is often reduced to ownership of an account at a financial institution. However, it is dangerous to think of this phenomenon in this extremely limited way. The problem of people and companies without access to the formal financial system, that is, unbanked, can be easily solved by making an account available at a financial institution, however, does this mean that they are now included in the financial system?

Although availability and access to financial institutions are necessary conditions for financial inclusion, this is certainly not enough (Quiñonez, 2017). In this context, it is clear that banking is only the first step towards financial inclusion.

There is no single approach to defining financial inclusion, but there are some characteristics inherent to the phenomenon, which are: (i) availability of institutions in the formal financial system; (ii) access to institutions in the formal financial system; (iii) regular use of products and services from the formal financial system; (iv) good quality of financial



products and services in order to enhance the well-being of society (Demirguc-Kunt et al., 2007; Kabakova & Plaksenkov, 2018).

Raichoudhury (2020) considered that it is a process that ensures that all members of an economy do not have difficulty accessing and making use of the services provided by the formal financial system. An inclusive financial system allows for a better allocation of productive resources, a reduction in capital costs, an improvement in the management of personal finances, and, in addition, it manages to stop the expansion of informal sources (such as loan sharks, for example).

The World Bank defined financial inclusion as the process by which individuals and companies have access to useful and accessible financial products and services that are able to meet their real social and economic needs, as payment instruments (for example, credit and debit cards, remittance transfer and receipt services), savings and investment accounts, consumer and/or investment credit, insurance products, among others (Demirguc-Kunt et al., 2018; Demirgüç-Kunt & Klapper, 2012).

Thus, financial inclusion can be summarized as the social phenomenon in which a wide range of people and companies are available to access financial institutions and make use of their products and services, which are offered in a fair and adequate manner to their real needs (Ahamed & Mallick, 2019; Allen et al., 2016; Bongomin et al., 2018; Demirguc-Kunt et al., 2018; Demirguc-Kunt & Klapper, 2012; Kim et al., 2018; Ozili, 2018; Oz-Yalaman, 2019; Swamy, 2014; Varghese & Viswanathan, 2018). In other words, this process begins with obtaining a transaction account, and is scaled through the use of payment instruments, savings and investment products, credit products, insurance products.

This work assumed the objective of building a composite index of financial inclusion for Brazil. This proposal is in line with the wishes of the World Bank, especially in its Financial Inclusion Support Structure (FISF) (World Bank, 2021b), an initiative that aims to accelerate and increase the effectiveness of reforms and other actions to achieve goals for the development of a more inclusive financial system.

The World Bank (2021b) it has the Country Support Programs (CSPs), which were organized into four thematic areas: 1) national strategy for financial inclusion, monitoring and evaluation; 2) financial infrastructure such as payment and credit reporting systems; 3) diversified financial services for individuals and businesses; 4) consumer financial protection



and financial standing. It is understood that the findings of this work can contribute to supporting CSPs.

It is worth mentioning, as explained by Shiller (2013a), the fact that the financial system needs to be thoughtfully forward-looking; more importantly, it must be expanded, democratized and humanized so that its institutions are even more diffuse in their reach and positive in their impact. This means giving people and businesses the ability to participate in economic life on an equal footing, so that they can actively and intelligently explore their opportunities.

The issue of financial inclusion has emerged as an issue of paramount importance for world development, becoming one of the main challenges placed on the agenda of international organizations such as the World Bank, the International Monetary Fund (IMF), the Inter-American Development Bank (IDB) and the United Nations (UN). It is also a topic of attention for many policy makers, private sector strategists, central banks and even nongovernmental organizations (Allen et al., 2016).

Financial inclusion is also present in at least eight of the seventeen SDGs of the UN 2030 Agenda (Klapper et al., 2016; Makina, 2019), specifically, SDG1 (on poverty eradication), SDG2 (on ending hunger), SDG3 (on profit and health and well-being), SDG5 (on gender equality), SDG8 (on promoting growth economic and employment), SDG9 (on supporting industry) and SDG10 (on reducing inequality). In addition to these, there is the SDG17 (on strengthening partnerships and the means of implementing solutions to achieve the previous SDGs).

An inclusive financial system necessarily involves the democratization of finance. Financial inclusion represents a social phenomenon that is characterized by the availability, access and use by people and companies of the products and services offered by financial institutions. Recognizing its importance for economic and social development, it is important to use instruments to measure its stage or magnitude in a given territory. This can be done through the analysis of individual variables related to the phenomenon or through the creation of representative indexes.

How a measure of financial inclusion is established depends on how the phenomenon is defined (Sarma, 2012). There are basically two streams of investigation in the empirical



literature, they are: (i) execute financial access variables separately; (ii) build composite financial inclusion indices, that is, multidimensional indices.

The first stream can be characterized from some works that seek to measure financial inclusion using the proportion of the adult population that has a relationship account with a formal sector financial institution. However, it is known that, currently, this type of research can only reveal the first step towards effective inclusion, which is banking. Furthermore, this type of measurement ignores other dimensions associated with the phenomenon, such as availability and usage. For example, Seidman, Hababou e Kramer (2005), analyzing the situation of low-income people in some North American cities, they managed to show that, despite two-thirds having accounts in financial institutions in the formal sector, they still ended up resorting to the informal sector to obtain credit, that is, even that a good proportion of that population can be considered banked, cannot make effective use of what is offered by formal institutions.

The other stream of research is intended to go beyond the simple measurement of bankarization in favor of a broader understanding of the phenomenon of financial inclusion. For this, they analyze access (banking) together with other dimensions, such as availability and use, thus having a multidimensional bias (Beck et al., 2007, 2009). It is in this line of investigation that this study fits.

For these reasons, the most suitable is to evaluate financial inclusion through the construction of a composite index, which is able to capture the multidimensionality of the phenomenon. Thus, the objective of building a composite financial inclusion index for Brazil was assumed. In addition, there is, of course, the construction of dimensional indices that are also representative of the phenomenon, in particular, an index of availability of the Brazilian financial system, an index of access to the Brazilian financial system and an index of use of the Brazilian financial system.

This study innovates by developing a measure of financial inclusion that incorporates, in its variable dimensions related to digital technology, something that has not yet been reported in previous empirical research focusing on the Brazilian territory. This paper is organized in four sections. After this introduction, we have the data and applied method, followed by the analysis of the results and final considerations.



2. DATA

The details of the data used to operationalize the wishes of this work are shown in the following Table 1. It is noteworthy that the time series considered extends from the year 2008 to the year 2019.

Table 1

Data used to build the composite financial inclusion index for Brazil

Identification	Variable	Source
Cmb	Mobile Banking Accounts [in millions]	Banking Technology Surveys of the Brazilian Federation of Banks
Tmb	Banking Transactions made via Mobile Banking [in billions of units]	(Febraban), years of (2013, 2014, 2015, 2017, 2018, 2019, 2020)
Cc	Current accounts [in millions]	Central Bank Time Series
Ср	Savings Accounts [in millions]	Management System (SGS -
Ag	No. of Bank Branches	Bacen) (2021)
Pae	Number of Electronic Service Points of Banking Institutions	
Cor	No. of Banking Institutions Correspondents	
Cpib	Bank Credit in relation to Gross Domestic Product (GDP)	
Ppib	Pending Savings in Banking Institutions in relation to Gross Domestic Product (GDP)	International Monetary Fund (IMF – data) (2021)
Ait	Internet Access for the Brazilian Population (10 years or more)	World Development Indicators (2021d) and Brazilian Internet Steering Committee (CGI) and Regional Center for Studies for the Development of the Information Society (Cetic.br) (2021)
PopulationBR (15+)	Brazilian Adult Population (15 years or older)	World Bank (2021a)
Companie-and-Others	Companies and other organizations active in Brazil	Directorate of Research, Central Register of Companies – Demographics of Companies of the Brazilian Institute of Geography and Statistics (IBGE) (2021).

Source: Authors

These variables were used to construct the composite financial inclusion index for the Brazilian territory. In addition, to make this possible, dimensional index were estimated, namely, the availability index of the Brazilian financial system, the access index to the Brazilian financial system and the use index of the Brazilian financial system.



3. METHOD

The process of monitoring, evaluating and promoting financial inclusion depends, to a large extent, on the adequate and systematic measurement of its dimensions. The construction of instruments such as indexes proves to be fundamental for observing how widespread the phenomenon is in a given territory, and also for drawing quantifiable goals and targets, which allow the assessment of strategies and possible necessary adjustment points.

Recognizing that the correct assessment of the financial inclusion stage in a territory must be made considering it as a multidimensional phenomenon, this study aimed to build a composite index for Brazil. For that, a multivariate method was used, namely, the Displaced Ideal Method (DI). The ID is based on the Euclidean distance to show the smallest distance of something from its ideal scenario (Zelany, 1974).

Euclidean distance is a relatively simple measure that can be explained as the length of a segment connecting two points, and that can be calculated from Cartesian coordinates using the Pythagoras theorem (Gower, 1985; Liberti et al., 2014). The Euclidean distance between points P and Q in an n-dimensional space can be defined as described by Equation 1.

Euclidean distance =
$$\sqrt{(p1-q1)^2 + (p2-q2)^2 + \dots + (pn-qn)^2}$$
 (1)

Although it is a very common measure, the Euclidean distance is not robust over a range of scales, which means that the computed results can be skewed if the units of the variables used have very different amplitudes. To overcome this problem, the most recommended is to use data normalization strategies.

The normalization of raw data can be done using the Min-Max formula, described in Equation 2, which performs a linear change in the original values causing the variables to be transformed in the range [0,1], which means that the minimum value will be 0 (zero) and its maximum value will be 1 (one). This ensures that they are all on the same scale. Again, it is known that the use of variables with different measurement scales can cause the model to suffer from estimation biases (Saranya & Manikandan, 2013).

$$X_e = \frac{X - Mi}{Ma - Mi} \tag{2}$$

Where: x_e represents the normalized variable; x is the current value of the variable; mi is the minimum value of the variable, that is, its lower limit, which is defined by some prespecified rule, but which normally equals 0 (zero); ma is the maximum value of the variable, that is, its upper limit, which is also defined by some pre-fixed rule.



This exposed, the ID assumes the existence of an interdependence between the representative variables of a phenomenon under analysis, arguing that the best possible system must have the shortest distance in relation to its ideal scenario. The method was proposed by Zeleny (1974), within the contextual framework of operational research, and aims to offer the determination of optimal solutions that support decision-making processes. Furthermore, it allows the identification of an optimal path that must be followed by a representative index until reaching its maximum point, known as the ideal scenario.

Specifically, in this study, the modified ID was used, which differs from the traditional ID by using the inverse of the normalized Euclidean distance. This is done to facilitate the interpretation of the results, since the constructed indices will present 1 (one) as an ideal scenario and 0 (zero) as a minimum value. It is noteworthy that other researchers have also used this modified method in their index construction work. you have the job of Sarma (2008), which sought to develop a composite financial inclusion index for India, using a set of variables related to the banking sector. The work of Sarma (2010), which used data from the World Bank and the National Monetary Fund (IMF) to build a composite index of financial inclusion for several countries around the world, focusing on the year 2004. There is the work of Gupte et al. (2012), which focused on the construction of a composite financial inclusion index for India for the years 2004, 2008 and 2009 Yorulmaz (2013), which built a composite financial inclusion index for Turkey. you have the work of Ambarkhane et al, (2016), which sought to develop a composite financial inclusion index also focusing on India, but considering variables related to the banking sector and the insurance sector. There is also the work of Sarma (2016), which developed an index of financial inclusion for several Asian economies, and that of Shen e Hueng (2021), which sought to develop a composite digital financial inclusion index for 105 economies around the world.

As noted, the use of modified ID is common in empirical studies aimed at building indexes. Also, as stated by Nathan *et al.* (2008), the method manages to satisfy six intuitive properties (axioms) that give it objectivity and robustness. These axioms are known by the acronym NAMPUS.

The first axiom is that of "Normalization" (N), which says that the index must have a minimum value (= 0 (zero)) showing no development in its dimensions. And it must have a



maximum value (= 1(one)) revealing full development in its dimensions. The two positions in an n-dimensional Cartesian space refer to the origin (= 0 (zero)), and the ideal (= 1 (one)).

The second axiom is that of "Anonymity" (A), which says that the composite measure is indifferent to the exchange of values between dimensions. This means that if the values of any of the dimensions are swapped between them, the index value remains the same.

The third axiom is of "Monotonicity" (M). This means that the index must be higher if the observed value in a dimension increase, even if the values of the other variables remain constant. The opposite is also true.

The fourth axiom is that of "Proximity" (P). This implies that the higher the index value, the smaller the distance from the ideal scenario.

The fifth axiom is that of "Uniformity" (U). The index must present a balance, that is, a greater dispersion in dimensions must result in a smaller value in the composite index. The opposite is also true.

The sixth axiom is that of "Signaling" (S). The index indicates a unique ideal path to reach the highest value. Thus, it not only suggests the current state of affairs, but also plays a futuristic role.

Once the axiomatic set of the modified DI method is exposed, we move on to the detailing of its estimation form, thus, the inverse of the normalized Euclidean distance in an n-dimensional space is obtained through Equation 3.

$$Index = 1 - \frac{\sqrt{(1-d1)^2 + (1-d2)^2 + \dots + (1-dn)^{\wedge 2}}}{\sqrt{n}}$$
(3)

Where $\sqrt{(1-d1)^2 + (1-d2)^2 + \cdots (1-dn)^2}$ represents the Euclidean distance, dividing it by \sqrt{n} for the consideration of n-dimensional space and then subtracting from 1 (one) the result, so that the values represent 1 (one) as the ideal scenario and 0 (zero) as a minimum scenario, considering that the raw data were normalized by the Min-Max formula.

Financial inclusion is an abstract concept that cannot be quantitatively measured directly. However, this can be done through the interaction of a series of variables. It is known that, behind a set of correlated variables, an underlying latent structure can be identified (Cámara & Tuesta, 2014). This is the essence of building a composite index.

The composite index representing the phenomenon of financial inclusion for the Brazilian territory built in this work has three dimensions: (a) index of availability of the



Brazilian financial system; (b) index of access to the Brazilian financial system; (c) Brazilian financial system usage index. These dimensions were chosen based on previous empirical studies, and specifically the works of (Ambarkhane et al., 2016; Sarma, 2008, 2010, 2012, 2016; Shen et al., 2021; Yorulmaz, 2013). Together, the dimensions provide a holistic view of the phenomenon under analysis, allowing one to see whether a financial system is inclusive or exclusive.

As explained in the previous subsection, the method selected to meet the objective was the modified ID; particularly, the modified DI in two stages was used. The first stage serves to obtain the dimensional indices, and the second stage aims to build the composite financial inclusion index for the Brazilian territory, useful to show how inclusive (or not) the country's financial system is, as well as to present its evolution in the period investigated.

The steps for modeling the DI method in the process of building the proposed indexes are as follows:

- (1°) Definition of the variables used in the construction process of each index;
- (2°) Normalization of raw data using the Min-Max equation;
- (3°) Application of equation 2 (DI method) using the normalized variables to obtain the dimensional indices;
- (4°) Application of equation 2 (DI method) using the values observed in the dimensional indices to obtain the composite index.

The dimensional index referring to the availability of the Brazilian financial system was constructed from four variables: (i) number of bank branches; (ii) number of electronic service stations; (iii) number of bank correspondents; (iv) internet access.

The dimensional index referring to access to the Brazilian financial system was built from three variables: (i) accounts with mobile banking; (ii) checking accounts; (iii) savings accounts.

The dimensional index referring to the use of the Brazilian financial system was constructed from three variables: (i) credit/GDP; (ii) savings/GDP; (iii) total transactions via mobile banking.

After explaining the variables considered in the construction of the dimensional indices, they are normalized, which was done using the Min-Max equation (Equation 2).



For the normalization of the variables "number of bank branches", "number of electronic service stations" and "number of bank correspondents", which were used to construct the index representing the dimension "availability of the Brazilian financial system", 0 (zero) and maximum values are those observed in the series itself. In situations where you cannot establish exactly the ideal scenario (maximum point), it is recommended to do this procedure (Saranya & Manikandan, 2013). For the normalization of the other variable that make up this dimension, namely, "Internet access of the population aged 10 (ten) years or older", the minimum value was 0% (zero) and the maximum value was 100% (one hundred).

For the normalization of the variables that were used to construct the index representing the dimension "access to the Brazilian financial system", the minimum value was considered to be 0 (zero) and the maximum value to the total Brazilian population in adulthood, population with 15 years or more (according to the method adopted by the World Bank), added to the total number of companies and other Brazilian organizations. This was done on the understanding that full access to the financial system is only achieved with the full banking of all adults, companies and other organizations.

For the normalization of the variables "credit/GDP" and "deposits/GDP" that were used to construct the index representing the dimension "use of the Brazilian financial system", 0% (zero) was considered as a minimum value and as a value maximum 100% (one hundred); nevertheless, it is recognized that these proportions can exceed 100% (one hundred) in some cases. For the other variable considered in this dimension, namely, "transactions via mobile banking" (in billions), the highest value that was observed in the series itself was taken into account as the maximum point and 0 (zero) as the minimum point.

After normalizing the variables, the first stage of the DI method is applied to obtain the dimensional indices. Specifically, the Equations (4, 5, 6) are arranged in the following ways:

Availability Dimension =
$$1 - \frac{\sqrt{(1-ag)^2 + (1-pae)^2 + (1-cor)^2 + (1-Ait)^2}}{\sqrt{4}}$$
 (4)

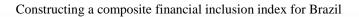
Where: ag (number of bank branches), pae (number of electronic service points of banking institutions), cor (number of bank correspondents) and Ait (access to the internet for the Brazilian population aged 10 (ten) or most).

Access dimension =
$$1 - \frac{\sqrt{(1-cmb)^2 + (1-cc)^2 + (1-cp)^2}}{\sqrt{3}}$$
 (5)

Where: cmb (mobile banking accounts (in millions)), cc (current accounts (in millions)), cp (savings accounts (in millions)).

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Dimension Usage =
$$1 - \frac{\sqrt{(1 - Cpib)^2 + (1 - Ppib)^2 + (1 - Tmb)^2}}{\sqrt{3}}$$
 (6)

Where: Cpib (credit/GDP), Ppib (savings/GDP), Tmb (transactions via mobile banking (in billions of units)).

After obtaining the dimensional indices, the second stage of the DI method is applied to obtain the composite index, with Equation 7 being expressed as follows:

Composite Financial Inclusion Index
$$BR = 1 - \frac{\sqrt{(1-D1)^2 + (1-D2)^2 + (1-D3)^2}}{\sqrt{3}}$$

(7)

Where: D1 represents the dimensional index representative of the availability of the Brazilian financial system, D2 represents the dimensional index representative of access to the Brazilian financial system, D3 represents the dimensional index representative of the use of the Brazilian financial system.

- The availability dimension refers to how easy it is to find a financial institution that offers banking products and services, both physically and digitally.
- The access dimension refers to the level of banking in a society (people, companies and other organizations).
- The use dimension refers to the effective enjoyment of financial products and services offered by financial institutions, especially in relation to transactions such as transfers, payments, collections, obtaining credit for consumption and investments, investments in remunerated accounts (such as traditional booklets of savings).

It is noteworthy that the data were organized using the Microsoft Excel software and were empirically treated with the help of the R software. Once the empirical modeling used was exposed, we move on to the next section with the results and analysis.

To interpret the results of the indices, both dimensional and composite, Guideline Note No. 18 of the Alliance for Financial Inclusion (AFI) (2016) is considered.

- $0.75 < index \le 1 = high level (high degree of financial inclusion);$
- $0.5 \le \text{index} < 0.75 = \text{level above the world average};$
- $0.25 \le \text{index} < 0.5 = \text{moderate level};$
- $0 \le \text{index} < 0.25 = \text{low level (high degree of financial exclusion).}$



4. RESULTS AND ANALYSIS

The following Table 2 presents the results of the dimensional indices and their respective descriptive statistics. In particular, three dimensions were estimated (availability, access and use of the Brazilian financial system).

Table	2
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Dimensional indices representing financial inclusion in the Brazilian territory

	D1 - Availability of the	D2 - Access to the			
	Brazilian Financial	Brazilian Financial	D3 - Use of the Brazilian		
Year	System	System	Financial System		
2008	0.52	0.30	0.26		
2009	0.59	0.30	0.26		
2010	0.63	0.32	0.27		
2011	0.69	0.34	0.27		
2012	0.72	0.37	0.27		
2013	0.75	0.40	0.27		
2014	0.76	0.44	0.28		
2015	0.77	0.45	0.30		
2016	0.77	0.46	0.33		
2017	0.80	0.55	0.35		
2018	0.81	0.59	0.37		
2019	0.81	0.65	0.40		
Average	0.72	0.43	0.30		
Maximum	0.81	0.65	0.40		
Minimum	0.52	0.30	0.26		
Standard deviation	0.09	0.12	0.05		

Source: elaborated by the authors.

As observed in Table 2, in relation to the representative index of the first dimension (D1), between the years 2008 and 2013 it presented results that indicate a level considered above the world average (values between 0.5 and 0.74) according to the AFI criteria. Between 2014 and 2019, it has a high level, which indicates that the financial system has a high degree of availability in the Brazilian territory (values between 0.75 to 1). Still, it is worth highlighting an evolution of around 0.29 units between the first and the last year of the series.

It is known that the simple availability of financial institutions does not guarantee effective access to the financial system, and, for this reason, the analysis of the representative index of the second dimension (D2), which can also be understood as the degree of bancarization of people and companies in a given territory. Between 2008 and 2016, there Revista Gestão & Tecnologia, Pedro Leopoldo, v. 22, n.1, p. 168-192, jan./mar.2022

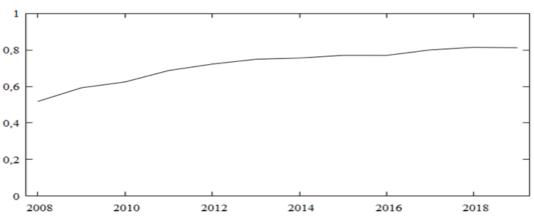


was a moderate level of access (values between 0.25 and 0.4). From 2017 to 2019, the verified level started to be considered as above the world average (values between 0.5 and 0.74). A relevant point is given by the fact that the values more than doubled between the analyzed period; specifically, there was an increase of 0.35 units. This may be due to the fact that, in Brazil, every citizen has the right to open a checking account in any bank free of fees, in accordance with Resolution No. 3,919/2010, established by the Central Bank of Brazil (Bacen, 2010).

Advancing in the analysis, it is worth noting that neither availability nor access to the financial system is capable of guaranteeing, alone or together, that people and companies make effective use of financial products and services. That said, we move on to the analysis of the third dimension (D3), whose results show that, throughout the period considered, the index presented a moderate level (values between 0.25 and 0.4). Despite this, an evolution between the initial year and the final year is recognized, with an increase of 0.14 units.

In summary, according to the interpretation criteria established by the AFI (2016), the availability dimension closed the series with a high level of inclusion, the access dimension (banking) closed the series with a level of inclusion considered above the world average, and the use dimension closed the series with a moderate level of inclusion.

Figures 1, 2 and 3 show graphs with the evolution of dimensional indices during the analyzed



period.

Figure 1 - Dimensional index related to the availability of the Brazilian financial system Source: elaborated by the authors.



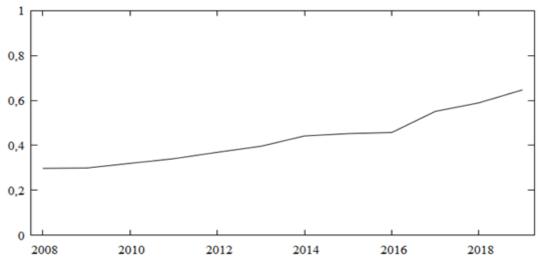
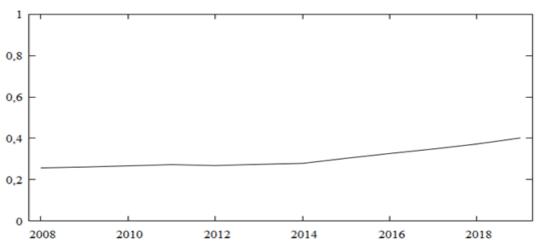
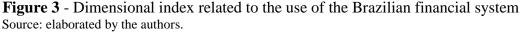


Figure 2 - Dimensional index related to access to the Brazilian financial system Source: elaborated by the authors.





We now move on to a more detailed analysis of the results, regarding the dimensional indices of financial inclusion (availability (D1), access (D2) and use (D3)). Although the first two dimensions showed satisfactory values at the end of the period (high level and level above the world average, respectively), the use dimension (D3) was presented at a moderate level. With this, it can be understood that even Brazilians, having financial institutions at their disposal and having access to them, are largely unable to make use of their products and services, therefore not taking advantage of the possible benefits that the system could granting it, such as reducing poverty, increasing income, consumption and savings, reducing various inequalities (gender, racial and economic), social and economic security (something that deals with aspects such as better quality food , health, education, various insurances, private ECCE Revista Gestão & Tecnologia, Pedro Leopoldo, v. 22, n.1, p. 168-192, jan./mar.2022



pension, among others), encouraging entrepreneurship, encouraging the habit of making investments, etc., as demonstrated by previous empirical studies, such as the works of (Demirgüç-Kunt & Klapper, 2012; Guiso et al., 2004; Karlan & Zinman, 2010; King & Levine, 1993a, 1993b, 1993c; Neaime & Gaysset, 2018; Park & Mercado, 2015).

This Brazilian context is very similar to the results identified by (von Fintel & Orthofer, 2020), authors who focused their analysis on South Africa, and showed that, despite the availability of the system and ease of access to it, its use is still restricted in that country. Specifically, the researchers identified that this may be due to the existence of credit modalities considered predatory for the most vulnerable population, since, despite the fact that financial inclusion has managed to increase the share of income of the middle class, it reduced that of the poorest population.

Remembering what was exposed by Shiller (2013b), the financial system is somewhat paradoxical, since while it facilitates some of the greatest economic advances, it can also cause certain social disasters. The solution to prevent these undesirable situations from occurring, according to the author, is through the release of financial innovations in all their aspects.

A possible path for the benefits of finance to reach a large portion of the population and companies can be seen through fintech-type institutions, which follow an operational logic different from the traditional model of financial institutions, offering products and services considered more sustainable and adequate to the socioeconomic reality of a territory. This, in a way, was identified in the work of Hodula (2021), which conducted a survey using a panel of 73 countries between 2013 and 2019, finding that fintech credit platforms can act as both a complement to and a substitute for traditional bank credit.

Hodula (2021) states that if more concentrated banking sectors invoke market inefficiencies, such as higher interest margins and slower procedures for credit operations, borrowers may migrate to fintechs. Another possible explanation for this migration may lie in the scarcity of credit offered by traditional institutions, causing a portion of the potential clientele to stay out of the banking supply chains.

In Brazil, it is known that banking concentration is relatively high. As shown in the Banking Economy Report of the Central Bank of Brazil (Bacen) for the year of (2020), the participation of the five largest banks in the country in the credit market is 68.5%, and the



percentage also takes into account financial institutions that include the non-banking sector, such as credit fintechs and credit cooperatives. In the case of the banking sector only, the percentage rises to 79.2% of operations. The report also presents data on the average interest rates in the market, which are at high levels, for example, the average interest rate on overdrafts, in December 2020, reached 127% pa, the average interest rate of free credit to families, in December 2020, reached 37.2% pa, and the average interest rate on free credit to companies, in December 2020, reached 11.6% pa Another fact that reinforces the finding of the sector in the country is related to the banking spread. According to figures presented by the World Bank, for example, since 2009 Brazil has been among the three countries with the highest rates in the world, in particular, since 2010 the country appears in second place, only losing to Madagascar, island located in the southwest of the African continent (World Bank, 2021c).

Now, we move on to the analysis of the composite index of financial inclusion index for the Brazilian territory. The results are shown in Table 3.

Year	Composite Financial Inclusion Index for Brazil		
2008	0.35		
2009	0.37		
2010	0.39		
2011	0.41		
2012	0.43		
2013	0.44		
2014	0.46		
2015	0.48		
2016	0.49		
2017	0.54		
2018	0.56		
2019	0.59		
Average	0.46		
Maximum	0.59		
Minimum	0.35		
tandard deviation	0.07		

Table 3:

Composite	index 1	referring	to financial	l inclusion	in the	Brazilian	territorv

Source: elaborated by the author.

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From the values shown in Table 3, it was observed that the Brazilian territory presented, between the initial year analyzed, 2008, until the year 2016, a moderate level of financial inclusion. As of 2017, financial inclusion in the Brazilian territory started to be considered at a level above the world average.

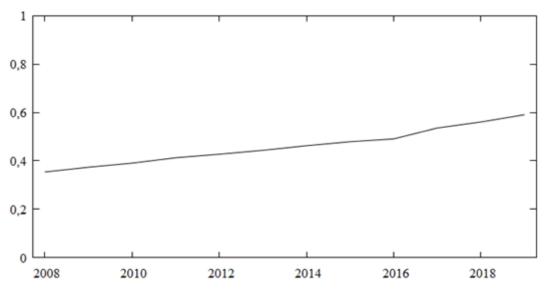


Figure 4 - Financial inclusion index for the Brazilian territory Source: elaborated by the author.

Comparing the results of the composite index with the values evidenced in the dimensional indices, there is evidence that allows the conclusion that the main barrier found for the advancement of financial inclusion in the Brazilian territory is related to the effective use of formal financial products and services offered by sector institutions. This may be due to their lack of adequacy to the reality of a large part of the population and Brazilian companies.

As exposed by Shiller (2013b), finance should not be seen as exclusive to a privileged social class, nor should it be seen as an engine of economic injustice; despite their inefficiencies and excesses, they represent a force capable of contributing to a more just and developed society, being a basic and indispensable social institution. However, the financial system must be oriented so that it is inclusive and adequate to encompass the greatest number of people and companies, that is, sufficient to meet the real needs observed in society.

The findings evidenced here can serve several purposes, including for political agents and monetary authorities to create strategies aimed at a readjustment of the Brazilian financial



system, so that it is sustainable and results in positive contributions to the country's economic and social development.

5. FINAL CONSIDERATIONS

An inclusive financial system reflects the democratization of finance, and this is increasingly important for a country's economic and social development. However, efforts to measure the phenomenon of financial inclusion are relatively scarce, especially in relation to the Brazilian territory; in addition, previous attempts did not consider in their estimates some variables from the digital world, something that undoubtedly contributes to measurement of failures.

This work assumed the objective of constructing a composite index of financial inclusion for the Brazilian territory, consequently, dimensional indices related to the availability, access and use of the financial system were developed.

In summary, the results show that, although Brazil has a good availability of financial institutions and also a high level of access, that is, banking, the use of financial products and services is still limited, which is one of the main barriers encountered. for the development of an inclusive financial system in the country. Hypotheses that can explain this were raised in the results and analysis section, and deserve further research to be, in fact, proven.

That said, it is understood that the study achieved what was desired, with the results likely to be used by various agents to create strategies aimed at building an inclusive financial system that is sustainable and positive for Brazilian society as a whole.

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